Considerations for Using Virtual Manipulatives & an Explicit Instruction Format in a Virtual Learning Environment

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Abstract

Providing effective mathematics instruction to learners with disabilities is challenging enough in a classroom. Providing the same level of high quality instruction can be more difficult in an online environment. This article describes an effective use of virtual tools for mathematics that can both engage students and provide the supports they need to learn the material. Throughout the article, the authors follow Tim's transition from teaching in his classroom at school to implementing his lesson plans through an online portal. A discussion of the concrete-representational-abstract methodology and explicit instruction adapted to use in an online environment guides recommendations.

Keywords: *Mathematics, Explicit Instruction, Concrete-Representational-Abstract*

Tim was in his fifth year as a special education teacher. Students came to him for math instruction. His classroom at the middle school was set up to provide the highest level of instruction to the ten students who came to his class. He taught a wide range of mathematics skills to the students, from basic numbers and operations to introductory algebra. He structured his instruction based on the learner needs and used an explicit instruction format. Tim's classroom was considered a Tier 2 intervention room in a Multi-Tiered System of Support adopted by the district.

Mathematics instruction is an important component of education for any student (Camera, 2015). It is one of the core subjects as evidenced by the standardized tests given to all students every year. Students in special education services often struggle with learning mathematics. They tend to enter school without certain prerequisite skills that allow them to learn even the most basic mathematical concepts (Izuno-Garcia et al., 2021; Mackintosh & Rowe, 2021). Early numeracy skills like simple number identification and counting provide the base on which to build more advanced mathematics skills. The National Council of Teachers of Mathematics (NCTM, 2000) recommended that all learners develop competent skills in reasoning and computation. Their Equity Principle recommended high expectations for all students. To achieve this, they suggest accommodating differences in learners through technology and other resources (NCTM, 2000).

Special education services often allow for different accommodations that provide the students with access to equitable levels of instruction (Every Student Succeeds Act, 2015). Some of these accommodations include the use of evidence-based practices and additional classroom staff who can provide prompting. This, however, is not always sufficient for students to develop mathematical skills. Providing instruction that allows the students to develop the necessary skills must include connecting the concepts to something in the learner's environment (NCTM, 2000). Without the context, students are less likely to develop understanding and rely on rote responding (Agrawal & Morin, 2016).

Evidence-Based Math Instruction Practices

Explicit Instruction

Explicit instruction is an evidence-based method used for mathematics instruction in special education classrooms (Strickland & Maccini, 2010). It is often used in conjunction with a Concrete-Representational-Abstract/Virtual-Representational-Abstract (CRA/VRA) methodology. Explicit instruction consists of modules that cover topics in three phases: modeling, guided practice, and independent practice (Doabler & Fien, 2013).

Throughout each phase, the teacher guides learners through the skill, providing support to both the entire class and individuals. This should also include the use of an "advanced organizer" that sets up the lesson for the students (Agrawal & Morin, 2016). An advance organizer can take many forms, from formal documentation to simple instruction, but generally provides the learners with information on how the new lesson is related to past lessons and provides rationales for the importance of learning the new skill (Archer & Hughes, 2010).

The modeling phase of explicit instruction involves the teacher working through the process of completing the problems with the students. Modeling includes both explaining the concept being taught and working through several examples of relevant problems (Archer & Hughes, 2010). This can also involve the use of manipulatives, either concrete or virtual. By demonstrating how to use the manipulatives while introducing the new concepts, the teacher can pair the two for the students and transition to the next phase of explicit instruction.

Guided practice comes after the students have seen several examples of the new concepts modeled by the teacher. The time it takes to reach the guided practice phase will depend on the student's abilities, quality of the modeling, and the complexity of the concept. If the teacher has done a good job of breaking down the skill in to small, manageable parts prior to modeling it for the students, learning that concept progresses faster (Archer & Hughes, 2010). In this phase, the students and teacher work through several examples of the concept together, stopping after each step to check for understanding and provide feedback to the learners. Manipulatives used in this phase can also aid the teacher's ability to detect problems with understanding.

Finally, teachers move to independent practice where they supervise the students as they complete work using the concepts they were just taught. This often takes the form of worksheets using problems similar to those discussed in the previous phases of the module. This type of formative assessment allows the teacher to see what types of mistakes the learners are making. Errors can be classified into one of three categories that can guide the teacher on error correction strategies (Stein et al., 2018).

Fact errors are easiest to correct by having the learners practice the basic facts that are often committed to memory, like addition and subtraction facts to 10. Component-skill errors come from understanding the steps in a strategy to complete a problem type, but completing the steps in the wrong order. For example, a student may know he needs to "carry" when adding numbers that sum greater than ten in a two-digit addition problem, but puts the 1 under the ones column and the other number above the tens column. Continued practice focused on this skill can eliminate this error. The most serious error detected in the formative assessment would be a strategy error. This occurs because the student has not actually learned the strategy and cannot complete any steps in the progression. If a teacher sees this type of error, they will need to reteach the strategy from the beginning.

Explicit instruction provides the students with all the supports they might need to learn math concepts. The graduated practice forms allow early informal assessment of learning and can lead to additional practice either with the entire group or focused on specific individuals. This type of instruction has traditionally been delivered in person, but can easily be adapted to an online presentation as well with few modifications.

CRA/VRA Instruction

Tim was particularly fond of all the manipulative items he used for teaching math. Tim liked to have his students use small objects for counting and completing simple math problems. The students enjoyed it as well. They could combine the small Unifix Cubes into larger sets when the numbers became larger. They could break off one or two when the problem called for subtraction. Tim's students were performing quite well on their math goals. That is, until the school closed down due to the pandemic. Tim now had to teach his students online. He had never used an online format and had only used a learning management system when he was in school. Incorporating his lessons into the virtual environment and still teaching to the students' goals was going to be new territory for Tim.

Special education teachers have found many different ways to introduce concepts in a way that provides context to the learners. One such way is through the Concrete-Representational-Abstract (CRA) learning methodology. The CRA methodology is an evidence-based framework of teaching that starts with students using some sort of manipulative device to provide context for the concepts being taught (Bouck et al., 2018). For example, a teacher might provide small blocks for students to use as they develop counting with one-to-one correspondence. The students would touch or move one block as they count aloud. The blocks can be used for teaching many different mathematical concepts.

As students develop competency using concrete objects, they are moved to the representational phase. Pictures are used in this phase through worksheets (Stroizer et al., 2015). For example, a problem would show the student pictures of 5 circles and 3 squares and ask the student to count the total number of shapes. The pictures of the shapes on the worksheet are representations of the actual objects. If students struggle with the conversion to representational, they can continue using concrete objects until they are more comfortable. This transition may take some time, depending on the learner.

Fading the use of the representational phase occurs when students no longer need any physical representation of numbers to complete problems (Stroizer, et al., 2015). This is the abstract phase of CRA. Once students reach this point in their ability to complete mathematic problems, they are ready for more abstract concepts. Students should be able to complete calculations in their head through fluency with basic facts (Flores, 2009).

The CRA methodology has a long history of use in classrooms. Many teachers provide items such as Unifix Cubes or Base 10 Blocks for their students throughout the concrete phase. However, there is an emerging use of technology that is replacing the actual concrete objects. Virtual objects, delivered on tablets or computers, are starting to be more common in classrooms (Bouck et al., 2018). Although these might seem to be more representational because they are not physical, they differ from the items used in the representational phase because they are easily manipulated, like concrete objects. Students simply click and drag the objects to different spaces on the screen as if they were picking up and moving a Base 10 block.

The digital age has provided alternative ways to use the CRA methodology. The number of websites offering virtual manipulatives is growing every year. Table 1 provides a list of the top sites in a typical Google search. Unifix Cubes, Base 10 Blocks, and color tiles are popular manipulatives for early numeracy skills such as building number sense or basic addition and subtraction (NCTM, 2000). These concrete manipulatives have been easily translated to a virtual world. Students are often introduced to virtual manipulatives through the use of SMART Board® technology or through the use of a projector connected to a computer. For example, SMART Board® technology does allow for the manipulation of the objects on the screen with simple hand gestures while using the computer requires the use of a mouse or touchpad (SMART Technologies ULC, 2021). With a SMART Board®, students can walk up to the screen and use their hands to engage with the manipulatives. This is similar to the way they interact with concrete manipulatives

Incorporating virtual manipulatives during large or small group instruction starts with the teacher modeling the use of the manipulatives. In an explicit instruction format, this first happens during the introduction of the concepts as the teacher demonstrates how to solve the problem. With the use of the SMART Board® in front of the class, the teacher not only demonstrates the procedures to solve the problem but also how to use the virtual manipulatives. The guided practice phase of explicit instruction requires the students to use the virtual manipulatives on their own. Table 1 (next page) provides virtual manipulative website information.

Table 1: ***Virtual Manipulative Website Information***

Website Name	Cost	Platforms	Customized Manipulatives	Ability to Write	Embed Ability	Numbers & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
Math Playground	Free + subscription	1, 2	Y	N	Google Classroom	Y	Y	Y	N	N
Didax	Free	1,2, 5	Y	N	Y	Y	Y	Y	Y	N
Mathigon	Free	1, 2, 3, 4	Y	Y	N	Y	Y	Y	Y	Y
Toy Theater	Free	1, 2	Y	Y	N	Y	Y	Y	Y	Y
DreamBox Learning	Free	1, 2	Y	Y	N	Y	Y	Y	Y	Y
GeoGebra	Free	1, 2	Y	Y	Y	Y	Y	Y	Y	N
ABCYa	Free + subscription	1, 2	Y	N	N	Y	Y	N	Y	N

Cool-math4kids	Free	1, 2	Y	Y	N	Y	Y	Y	N	N
Visnos	Free	1, 2	Y	N	N	Y	Y	Y	N	N
The Math Learning Center	Free + subscription	1, 2, 4	Y	Y	Y	Y	Y	Y	N	N
Mathsbot	Free	1, 2	Y	N	N	Y	Y	N	N	N
PHET Interactive Simulations	Free	1, 2	Y	N	Y	Y	Y	Y	N	N
National Council of Teachers of Mathematics	Free + subscription	1, 2, 3, 4	Y	Y	Y	Y	Y	Y	Y	Y
National Library of Virtual Manipulatives	Free	1, 2	Y	Y	Y	Y	Y	Y	Y	Y
Brainingcamp	30 day trial/Subscription	1, 2, 3, 4	Y	Y	Y	Y	Y	Y	N	N

Note: 1 = Mac; 2 = PC; 3 = Android; 4 = iPad; 5 = Chromebook

This may require individual tablets or computers for students. The students could also take turns coming up to the front of the class and using the manipulatives on the board under the guidance of the teacher. Because learners today are generally fluent in the use of handheld devices, prompting students to find the website for the virtual manipulatives would likely be the only further instruction required. If the teacher has also modeled how to find that during the first phase, the students should be able to do it independently. Additionally, the teacher may be able to set up the device so the only icon available for the student will lead to the virtual manipulatives.

Tim's students enjoyed their time on the computer at school. They would listen to audio books or play games. Tim used his classroom SMART Board® *to present lessons during class. The students liked to come to the board and swipe to get the correct answer or draw numbers. Technology had been a valuable asset in Tim's classroom. But now the students were at their own homes and Tim was teaching them remotely—through the computer—instead of with a computer. He could not use his Unifix cubes to help them with math problems. He and his paraprofessional could not prompt the students to count correctly. Students could just walk away from the computer or close out the video window. How was Tim going to help the students meet their math goals?*

Universal Design for Learning

Universal Design for Learning (UDL) is one framework for instruction that can be used to engage students effectively in both an in-person and an online format (CAST, 2018). UDL guidelines are meant to create learners who are motivated, knowledgeable, and goal-directed through creating means of engagement, representation, and action and expression (CAST, 2018). Basham et al. (2020) suggested UDL can improve outcomes for learners in special education. UDL encourages teachers to design the lessons to allow the students to easily access the content and create a sense of understanding through contextual practice. When students can interact with the materials while learning the concepts, they can gain both procedural and conceptual knowledge of those concepts with practice.

In mathematics instruction, students often struggle because of the presentation of the material. The concepts are generally presented with text and numbers spread across worksheets (Hall et al., 2012). This can lead to confusion about what stimuli is important for the problem. Simplifying that presentation can make the salient points more important. Attending to the relevant stimuli is also critical when teaching in a virtual environment. One advantage that virtual learning can provide here is actually the limitations of what can be conveyed by video.

Video modeling, an evidence-based practice used to teach many skills in classrooms and clinics, allows the instructor to focus the lesson on specific aspects of the skill and filter out non-essential parts that could confuse the learner (Park et al., 2019). Bellini and Akullian (2007) suggested after reviewing research on it that video modeling may be successful for learners with autism spectrum disorder because of the ability to remove non-essential stimuli.

This principle also applies to virtual learning. Teaching in a virtual environment where the camera limits what you might be able to show the learners, focusing attention becomes easier. Planning the lessons takes on more importance in this environment. Using the principles of UDL to create the lessons increases the likelihood that students will be engaged with the material. As mentioned earlier, UDL works by creating multiple means of engagement for the learners, different representations for the material, and multiple ways for the students to demonstrate their knowledge (Hall et al., 2012). Each of these three guidelines can be achieved using an explicit instruction methodology for the lesson.

Virtual Teaching Environments

Learning Environments

Virtual learning environments are becoming more ubiquitous as technology improves and permeates society. New online schools are providing a complete educational experience by delivering content through designed websites and learning management systems (LMS). According to Molnar et al. (2019), as of 2019, a total of 501 virtual schools across the United States were providing full-time education to nearly 300,000 students. The global COVID-19 pandemic has accelerated this pattern. Schools that were once delivering in-person instruction were required to quickly switch to an online delivery method. For many teachers, this switch was difficult and affected the way they provided instruction. The materials they used in class were left in the building and not available. Students were unlikely to have their own materials at home, especially in poorer households.

ZOOM or WebEx platforms are often being used to bring the teacher to the students. Other LMS can provide more structure than simple video conferencing platforms. However, an LMS may not provide opportunities to interact with the student, acting only as a host for material. Table 2 provides examples of some common LMS with relevant features. Figure 1 shows the inclusion of a link to a manipulative website as part of an assignment in Google Classroom.

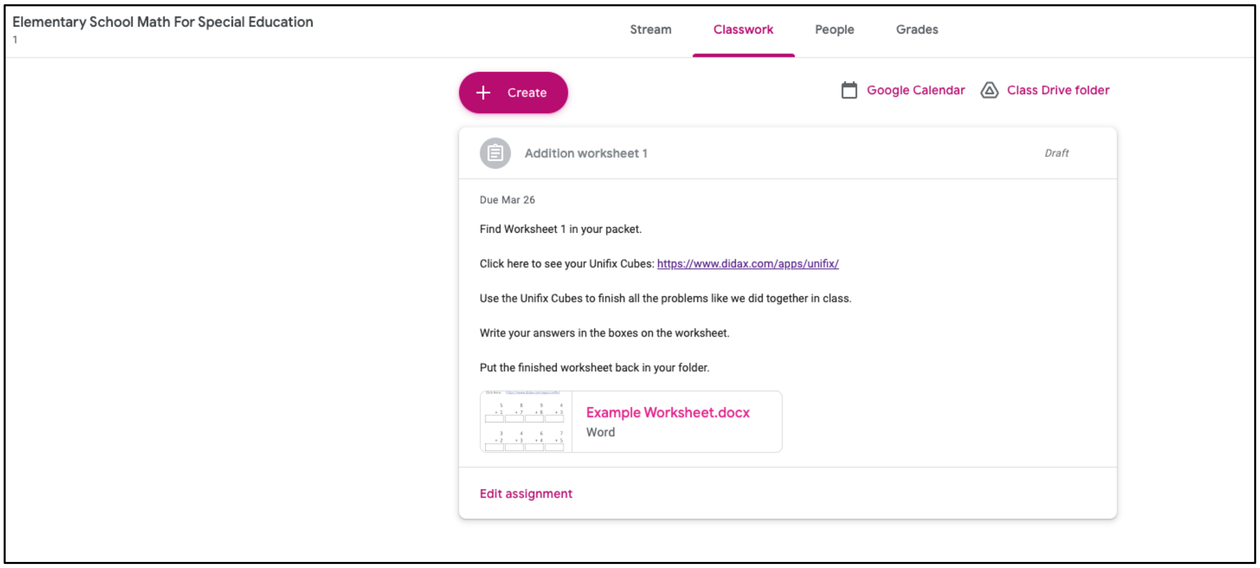
Table 2: *Examples of Learning Management Systems*

Name	Chat/Discussion Box	Share Screen	Share Links
BlackBoard	Y	Y	Y
Canvas	Y	N	Y
Google Class room	Y	Y	Y
Its Learning	Y	Y	Y
PowerSchool	Y	N	Y

To successfully reach their special education students, teachers must create appropriately engaging material. Explicit instruction in mathematics, regardless of location of the instruction, provides that engagement with the students since two of the three phases involve the students working with the teacher to solve problems. Engaging students using a virtual platform may seem to present difficulties in delivering the material, but the teacher can effectively reach them using the active responding that is found in explicit instruction. As Table 2 showed, common LMS have a chat feature and students could use the computer's microphone to respond to teacher queries.

As Tim was trying to put his lesson plans online, he realized he needed something to replace his concrete manipulatives for the students. He used virtual Unifix cubes from the Didax.com website with his SMART Board® *in the classroom, so students had seen them in action. They were just like the concrete cubes the students used in class. He decided to include a link to the website with the material he created for the lessons. Students could simply click on the link to open the website and use the Unifix cubes just like Tim did in class. Tim created several weeks' worth of worksheets and related material for each student and sent them to the student's homes. This was important because he could not include everything in the LMS and did want to have some permanent products of student responding. Students would be responsible for returning the worksheets after completion.*

Figure 1: ***Sample Assignment Screen From Google Classroom***



Using virtual manipulatives when teaching in a virtual environment does not have to be significantly different from that of the classroom. When using an explicit instruction framework, manipulatives can easily be integrated into the lesson using the virtual learning environment. Providing a link to your students for the type of manipulative needed for a specific problem can alleviate issues with students having access to the right material. During a specific session, the student may have to toggle between screens when using virtual manipulatives. Having the knowledge to do that or someone able to prompt that behavior is very helpful. For modeling and guided practice, the students should observe the teacher using the virtual manipulatives. This will allow them to see how they work before using them on their own.

Synchronous vs. Asynchronous Instruction

Teaching in a virtual environment provides the instructors with multiple ways to reach the students. There are two ways to provide instruction. Each has its advantages and disadvantages for both the teacher and student. The first method is synchronous instruction. It is called synchronous because the teacher and student are online at the same time and interacting directly. Amiti (2020) considers this to be the most important advantage. When using one of the LMS, including those described in Table 2, the teacher and students interact together as if they were in a classroom together. This is beneficial when using explicit instruction since the format requires modeling and guided practice.

Feedback can be immediate in a synchronous instruction format. The teacher can model the problems, check for understanding, and repeat as required. For the students, synchronous instruction offers the opportunity to ask questions in real time. It may be more difficult for the teacher to see the student's work, but if the student is able to either hold the work up to the camera or explain it to the teacher, it can approximate the same feedback that could be received in the classroom. A disadvantage for students with synchronous instruction is that they must be available at a certain time of the day and that time may not be possible when sharing equipment with siblings or other family members. With multiple family members working from home and limited resources, synchronous instruction can be difficult to schedule. Recording a synchronous lesson can allow students that have scheduling problems view the lesson at a later date.

The advantage of asynchronous instruction is that it allows the students to view the lesson at their leisure (Sadeghi, 2019). It removes the barrier of sharing resources with other family members. Calder et all, (2021) noted when family members need to share resources, such as parents that are working from home, educational use of those resources often takes lower priority. Asynchronous learning can alleviate possible congestion in homes with shared resources. Asynchronous learning can also be advantageous for students that may need additional prompting from parents or caregivers that would not be available during a scheduled class time. However, it does not allow the teacher to identify any student errors during the lesson itself like in an in-person or synchronous online lesson (Amiti, 2020). Thus, errors can only be identified after the lesson is completed and turned in. Finally, asynchronous learning can be more labor-intensive for teachers. Using this format, the teacher creates a video of the lesson and uploads it to the LMS prior to the lesson. The lesson can generally be activated by clicking on a link.

In either case, students need to log in to the LMS and attend to the material being presented. This can be problematic. Simply logging into the LMS does not guarantee the student will view the lesson or that they will attend to the material. Amiti (2020) and Sadeghi (2019) both list distractions as disadvantages of online learning. Self-directed students do well with online instruction. Students requiring consistent prompting typically struggle in the online environment. Attending to something that may be non-preferred for an extended period of time can be difficult for some learners. If a family member or caregiver is not available to prompt attention to the material, the student is likely to wander off and miss instruction.

Barriers to Instruction

In a Pew Research study, about 15% of students reported not having internet access at home, with more than 40% of learners of minority status and lower incomes reporting no access (Anderson & Perrin, 2018). Nearly 50% of teens in the study from households with an income less than $30,000 per year reported they often used a cell phone to complete homework. These figures do not describe specifically the impact on students with disabilities. However, Schifter et al. (2019) found in a small scale study that families from lower income brackets were more likely to be placed in special education programs than those with higher income. They defined “lower income” as qualifying for free or reduced lunch programs which places the families near the poverty line. Schifter et al. only examined special education placement as a function of income and did not break down demographics in any way. However, Gordon (2017) had reported that the United States Department of Education suggested 36% of black children live in poverty as compared to 12% of their white counterparts in a 2016 report on disproportionality. Similar results were shown for Hispanic (30%) and American Indian (33%) children. This, however, is a discussion for another paper.

The availability of internet is not the only issue of access for students. When family members quarantine together, multiple people will need to use the computer, tablet, or phone for their own reasons. Parents working from home will need access. Siblings attending virtual school will also need access. If there are not enough devices available, someone will not be able to attend synchronously. Some schools have the ability to let students have a laptop to use at home, but not all districts have this type of resource available. When equipment availability is an issue, asynchronous instruction is the best option. The decision whether to provide synchronous or asynchronous instruction is ultimately determined by the school and teacher, but the needs of the students should be part of that discussion.

Another issue that might arise is the learner's ability to manipulate a computer. Learners with more severe disabilities may not have either the physical dexterity to maneuver a mouse or trackpad or the skillset to navigate the computer systems. Virtual manipulatives requires some amount of fluency with clicking and dragging using a mouse or trackpad. Devices with a touch screen may be a better option for learners with dexterity issues for touching and dragging a virtual icon. In the traditional classroom, it is often the function of an instructional assistant (e.g., paraprofessional) to provide the prompting necessary for these students to engage with the material. This is difficult to provide at home if parents or another caregiver are unavailable.

Students in special education often struggle learning in a virtual environment (Kim & Fienup, 2021). This can be for various reasons, but occurs often because they require additional support, such as a paraprofessional, to keep them on task and engaged with the material. In a typical classroom, the paraprofessional or teacher also provides additional prompting when needed. This may not be available at home, unless a family member or caregiver is with the student during instruction time. Without that support, the student is less likely to be engaged or even log into the online platform.

Satsangi et al. (2021) found that video modeling was beneficial for learners with specific mathematics learning disorders. This type of instruction can be inserted into either synchronous or asynchronous online learning. Bandura and Walters (1977) suggested that modeling can be an effective method of instruction. Video modeling is an extension of that idea. In an online environment, modeling is delivered as video models. Bandura and Walters (1977) found that attending to the material is a critical reason that learners modeled behavior as part of his social learning theory. Modeling more examples can offset this issue somewhat but the teacher risks the student tuning out if they are required to just watch for too long without being active in some way. The models do not have to be live (Grusec, 1992) but should motivate the learner to engage and model.

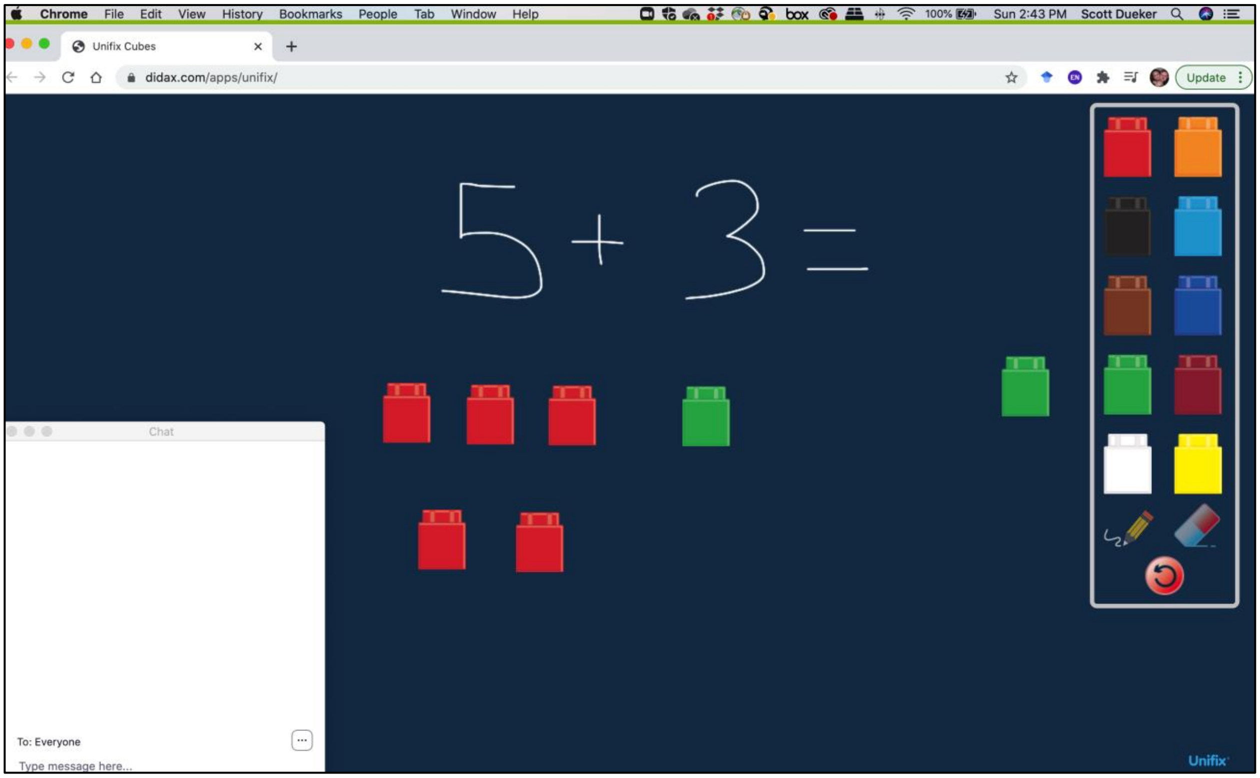
Another problem that might arise is that students in special education often adhere to rigid schedules. Particularly with students diagnosed with autism spectrum disorder, the schedules provide structure for the child and allow them to better integrate with others without presenting problematic behaviors (Rodger & Umaibalan, 2011). This rigidity, however, can lead to problems for the student and family when routines are disrupted, as in the case of COVID-19 quarantine and the move to virtual schools.

Students who were expecting to leave the house to attend class are now staying home. Because school and home are two separate spaces, the student may have difficulty with adjusting. The computer the student normally uses for fun activities is now used for classwork. This can lead to problem behaviors and poor attending. If the lesson is done synchronously, it may be able to fit into the normally scheduled time for the student. Asynchronous lessons might be more of a problem for students who struggle with schedule changes.

Considerations for Using Virtual Manipulatives

In his lesson on single-digit addition, Tim followed his typical explicit instruction format and modeled how to follow the addition task analysis. This included showing his students how to use the virtual manipulatives website. He would draw an equation on the screen, drag the items over to *the numbers, and drop them underneath (see Figure 2). Then he counted the objects to solve the equation. Just like he was using his Unifix cubes in the classroom. But he was on one end of the computer and his students were on the other. How could he get them to use the virtual manipulatives?*

Figure 2: ***Sample Shared Screen Working A Problem***



Virtual manipulatives can work well in a virtual instruction environment. Because the rest of the lesson is presented virtually, using the manipulatives must be as well. Incorporating virtual manipulatives into in-person lessons makes the transition to virtual learning easier, however, it is not necessary. As mentioned earlier, using UDL in lesson planning allows the teacher to engage the students in ways that will promote learning. Explicit instruction promotes the three foundations of UDL: multiple means of engagement, representations of the material, and actions (CAST, 2018).

Multiple Means of Engagement

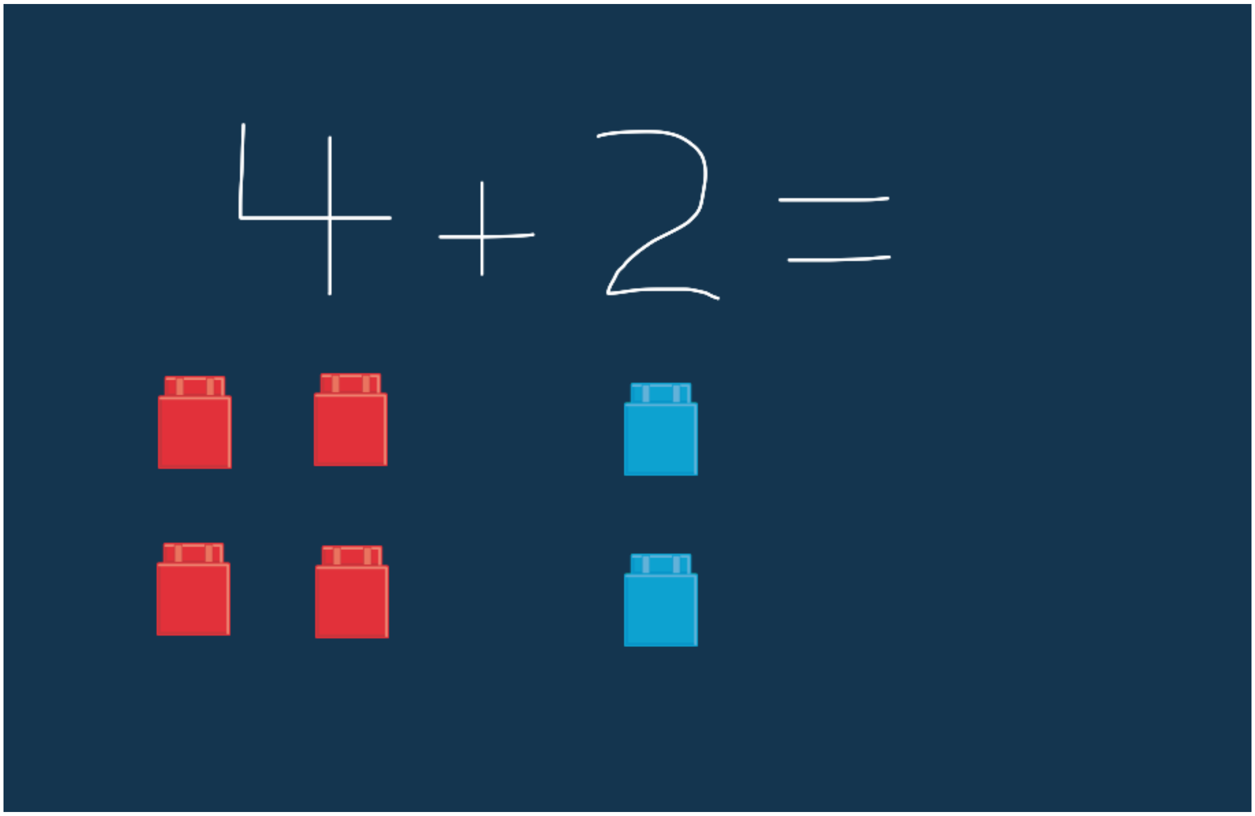
The three main components of explicit instruction are: modeling, guided practice, and independent practice (Archer & Hughes, 2010). These three parts engage the students across the lesson, from beginning to end, while learning the materials. The teacher will typically provide some background for the lesson using an advanced organizer and then move into the three areas of explicit instruction. Below is a simple task analysis for creating an effective lesson plan in an explicit instruction format. The task analysis provides the engagement through the three formats of UDL.

- Create several different versions of worksheets for the students
- Create a packet with worksheets and related material for each student
- Enter the lesson plan into the Learning Management System in three phases
    - Teacher-led examples
    - Guided practice
    - Independent practice
- Explain the rationale for the lesson and how it relates to previous lessons
- Demonstrate multiple examples of the new skill to the students
    - Use virtual manipulatives when appropriate
    - Conduct multiple checks for understanding
    - Provide feedback
- Guided practice
    - Using worksheets provided in the packet
    - Incorporate virtual manipulatives into problem solving
    - Monitor progress and provide feedback

As the teacher begins the lesson, the students become engaged in the passive mode of listening while the teacher both explains why the material is relevant and how to complete the problems. Learners in special education may have difficulty with the lecture format in this initial presentation. With nobody to prompt them to attend, the students may wander away from the screen or engage in other activities they find more reinforcing. Using a fast pace with required responses, even just asking for nods, may help teachers maintain engagement in this phase. During the phase, the teacher will also introduce the virtual manipulatives and demonstrate how to use them.

The phase of guided practice, the teacher will walk through solving problems with the students. Using worksheets provided, the teacher and students will methodically work through the steps of completing the new concepts. This will include using the virtual manipulatives when appropriate for the problem. Figure 3 shows an example of what a simple addition problem might look like using Unifix cubes. Teachers should ask to see the worksheet and how the student solved the problem at each stage of the solving process. This can be as simple as the student holding up the worksheet to the camera or asking the student to verbalize the process they used. The abilities of the students will determine which method to use. Feedback is an important part of the guided process. Solving multiple problems together will provide the students with enough confidence to move to the independent practice phase. It will also give the teacher some knowledge of the types of errors the students are making.

Figure 3: ***Example Problem Using Virtual Unifix Cubes***



Independent practice follows the guided practice phase. This phase can either use a new worksheet or the students can finish off the problems remaining on the worksheet from guided practice. During virtual instruction, independent practice can appear to be *dead time*. The teacher is not providing any instruction. The students are theoretically working on the problems on the worksheet. However, the teacher may respond to student questions. If all students are in the same virtual room, they may all hear the question and response. The teacher may move students to breakout rooms, depending on the LMS. This would provide some one-on-one space for the teacher and student. This phase provides the teacher with a means for evaluation of student learning.

Multiple Representations of Material

Embedding multiple representations allows the students to engage with the material in ways that can provide context. First, the teacher provides examples of the problems on screen. This could be simply pointing the camera at a worksheet or using a white board. White boards are useful for the initial phase of modeling the concepts. The teacher will demonstrate several examples of the problem solving process. In a typical in-person explicit instruction lesson, the teacher would solve two or three problems before moving into the next phase. In a virtual teaching environment, adding a few more could be beneficial for the students. The extra problems reinforce the concepts being taught and may act as additional prompting that may not be provided in the virtual environment.

Virtual manipulatives embedded into a lesson allow the teacher to bring clarity to the computational concepts. In the first phase of explicit instruction, the teacher demonstrates the use of the virtual manipulatives. The teacher should have a link to the virtual manipulatives webpage available on the screen. By clicking on the link, the teacher is providing a visual of how the student's screen will look when they click on the link. The modeling phase will show how to manipulate the objects on the screen and is included as an additional part of instruction. Teaching the students to use the manipulatives will require additional instructional time if the students have not had any practice in the regular classroom. Because virtual manipulative look and respond in much the same way as concrete manipulatives, students with experience with concrete manipulatives should ideally transition well.

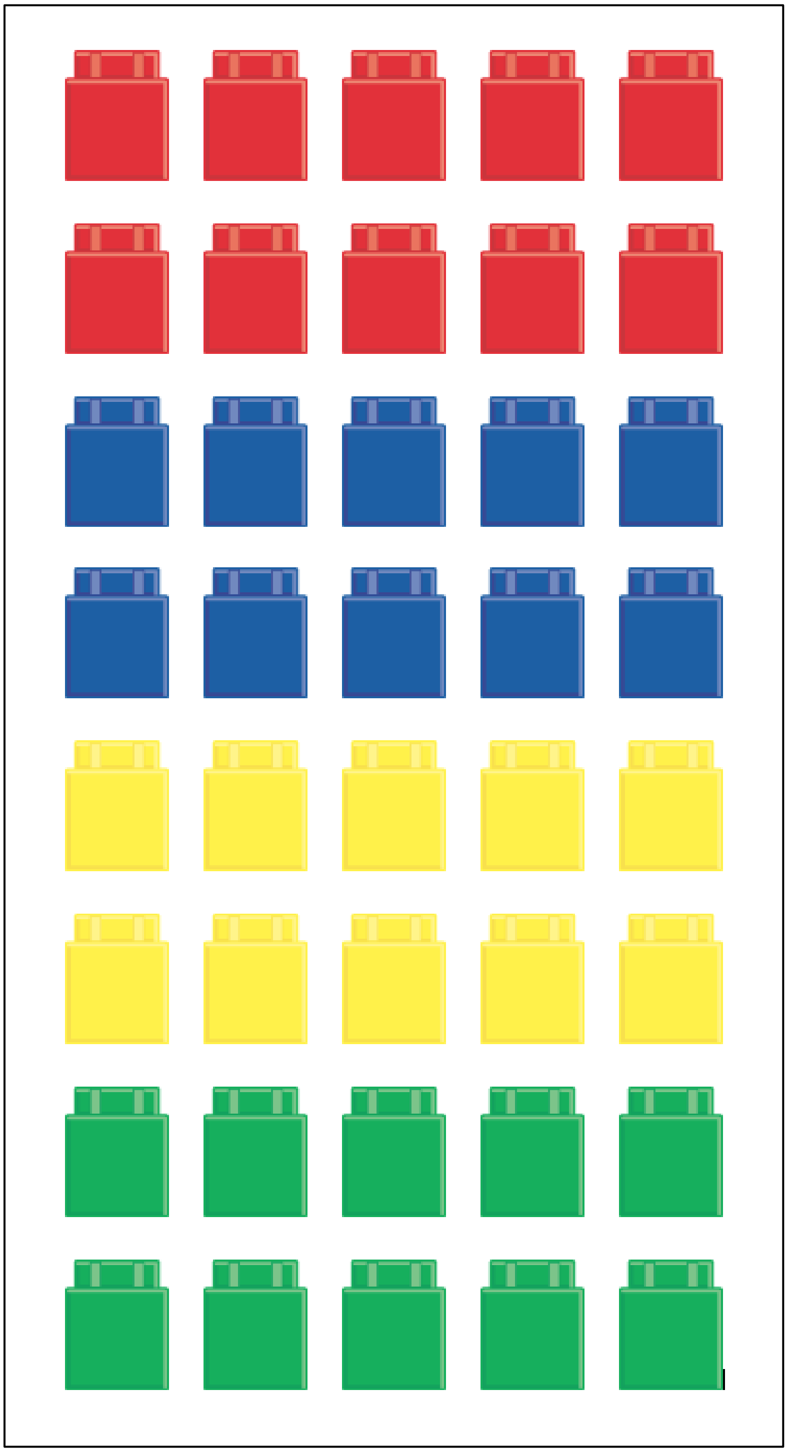
By providing problems using both worksheets and manipulatives, the teacher is creating context for the students. They will typically see the representations of the numbers in everyday life. Worksheets allow multiple examples of the skill being taught. The virtual manipulatives allow the student to view the skill in some context of real life.

Multiple Actions

Teaching is often described as the "sage on a stage" where the teacher stands up in front of the class lecturing, hoping the lesson takes hold (Morrison, 2014). Not all learners are able to be successful in this type of classroom. Students in special education, especially, need more active participation to learn the material. Engaging in a UDL framework and providing explicit instruction provides active responding. As discussed previously, when following an explicit instruction format, the teacher will engage the students in each of the three phases. Active participation provides the students with the opportunity to experience the lesson rather than passively listening, which may be a problem for students with attending issues.

During the initial phase of explicit instruction, the teacher models how to complete the work. This should not be simply lecturing. In a classroom, the teacher would provide students with the same materials being used during the modeling phase. Students would imitate the same movements as the teacher when completing the problems. This active participation sets the stage for guided practice to follow. In a virtual environment, this is a little more difficult. Students may not be able to manipulate the virtual manipulatives on the screen at the same time as the teacher while still maintaining presence on the LMS. Even if the teacher provides a link to the virtual manipulatives website, the student would be directed to a page other than the LMS and lose contact with the teacher. Alternatively, the teacher could ask the students to draw what they see on a piece of paper. The teacher could also include pages of representational pictures (see Figure 4) the students could mark as the teacher manipulates similar items on the screen. This action of imitating what the teacher is modeling helps keep the students engaged.

Figure 4: *Sample Representational Figures Worksheet*



With guided practice, the students should be solving the problems under the direction of the teacher. In a virtual learning environment, students should likely be using the representational aspect since they would not be able to use the virtual manipulatives for the same reason mentioned above. However, the teacher may use the virtual manipulatives in working the problems with the students. Both the teacher and the students would work from a common set of problems, provided in the packet. The teacher can include representational materials for each problem. In the classroom, guided practice could include 2-to-3 problems. Because the teacher is not able to be as attentive to the students in the virtual environment, working through more problems would provide the students with additional opportunities to practice and engage with the material.

Once the teacher feels the students have a grasp of the material, the lesson moves to independent practice. This additional action requires the students to complete worksheets of problems on their own. This phase may or may not be completed during the active instructional session. Worksheets may be provided in the instructional packet provided to the student. Another option would be for the teacher to embed the worksheet into the LMS. If the teacher does this, the student will have to navigate switching between screens to use the virtual manipulatives and accessing the worksheet, which may require prompting from a family member. If the worksheet can be completed outside normal instruction time and is not part of the LMS, the student may easily access the virtual manipulatives website. Once the worksheet is complete, the student can put it back in the packet and return it to school for grading. The teacher may also ask that the student hold the worksheet up to the camera to allow more immediate feedback and error diagnosis.

Conclusion

Tim finished the semester virtually. His were active participants in the online lessons. The explicit instruction format engaged the students. Tim often called on students to check their understanding and provided feedback when needed. His students used the virtual manipulatives from the link he provided them with the written materials. Even though he could not see them use the manipulatives directly, he could tell from the homework they returned to him that they were using them to successfully complete the work. Tim also recorded his classes because two of his students could not attend during the regularly scheduled time. He made himself available through email and chat to answer their questions as well. All of his students met their goals for the year despite the major changes in their class. Tim believes he will continue to use the virtual manipulatives in his classroom when everyone returns to in-person instruction.

Virtual instruction can be an effective method for teaching mathematics. By adopting a UDL approach and explicit instruction method, teachers can provide high quality opportunities for student responding. It can engage the students in ways similar to in-person teaching can. Each of the phases of explicit instruction requires many different actions on the part of the students. These actions include multiple methods for response and use of different tools. Virtual manipulatives can feature prominently in each phase when a lesson is developed. The teacher should demonstrate their use in modeling the lesson and for guided practice. This allows the students to see how to use them on their own during independent practice. The use of virtual manipulatives in the online environment creates the procedural knowledge that the students need to move to conceptual knowledge and greater understanding of the material. Effective and engaging lessons can be part of the virtual environment when lessons are designed using active responding with virtual manipulatives.

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